RESEARCH ARTICLE **(Open Access)**

Using the Cobb Douglas production function to find the optimal structure of tomato production in greenhouses

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Abstract

The agricultural sector is considered as the most important sector, which affects the development of the Albanian economy. The proposed reforms have been based on the need to increase transparency, accountability, effectiveness and efficiency in performing tasks in the agricultural sector. The government's attention should be focused on support of competitiveness of Albanian products, taking measures to improve the domestic market. This would reduce imports and increase exports. Among the key elements to stimulate the agricultural sector is to increase the production capacity in greenhouses. Greenhouse production should focus on those commercial products, such as the cultivation of vegetables or potatoes, which our farmers produce with less expense than other countries. This study analyzes the economic impact of some factors such as fertilizers, pesticides, irrigation, etc. and production. The study extends to 16 farms in the Lushnja region and focuses on the activity of tomato production in greenhouses. We used Cobb-Douglas production function to analyze the influence of some inputs in one output(tomato production). We have built the Lagrange function (LC) to minimize the cost of tomato production. An important conclusion of this study is that maximum revenue and profit maximization reached at the same point on the expansion path where the cost is minimal.The optimal structure of the tomato in this region was found using the necessary conditions for the minimum of LC.

Keywords: optimal structure; Cobb-Douglas function; tomato production; Lushnja region.

1. Introduction

Albanian agriculture is one of the most important branches of economy. According to Albanian Ministry of Agriculture and Rural Development, the progressive growth trend in exports is registering record figures, mainly due to the external high demand for Albanian fresh fruits and vegetables, including tomatoes, cucumbers, peppers, citrus etc. Meanwhile, the total volume of Albanian agricultural trade in 2017 amounted to 145.7 billion lek, registering a growth of 8.2 percent compared with 2016. Currently, the Agriculture Ministry has approved a fund worth 20 million euros that will be used to subsidy farmers and increase agricultural exports. One of the most important vegetables, with great nutritional value, curative in agro-processing industry is the culture of tomatoes. Our country has a favorable climate for the cultivation and production of high quality tomatoes. Tomato culture is widely used and occupies a considerable extent on the domestic market. Lushnja district is the area with the largest production of vegetables, which supply most of the domestic retail market. In the absence of safe market, the benefits of local farmers remains to be very low compared to other countries in the region. State Planning and intervention can contribute to increase profits of farmers in vegetable greenhouses. The main element of the stimulation in the agricultural sector is the increasement of production capacity in greenhouses, not only support for subsidizing fuel but also stimulate new technologies that reduce the cost of heating. Expanding areas of greenhouses is an absolute priority for increased production. In the last decade the cultivation of tomatoes is increased significantly, especially in protected areas (greenhouses), which improves the average yield for the cultivated area. Compared with some countries, the tomato yield per unit of surface and the amount produced is still small. The import of this crop from different countries is high. The improvement of the production technology, the increasing production areas and the use of different hybrids can reduce the import of tomatoes.

2. Material and Methods

Lushnja was the region where the target group were farmers engaged in greenhouse tomato production. After the discussion with experts the most important factors affecting the yield of tomato production were identified like organic manure, fertilizer, liquid fertilizer, pesticides and irrigation. To analyze the performance of production factors into the study, it was analyzed and implemented the factorial mathematical model:

$y = a_0 x_1^{a_1} x_2^{a_2} x_3^{a_3} x_4^{a_4} x_5^{a_5}$

where $x_j \; for \; j = 1, 2, 3, 4, 5$ are production factors.

While a_i for $i = 1,2,3,4,5$ are the parameters of the model.

These parametres are calculated using the average advanced method.

The calculation of parameters $a_i$performed with the formula:

$$a_i = p_i mes_p \left\{ \frac{\log \frac{y_k}{a_0}}{\log x_{ki}} \right\}$$

where $mes_p \left\{ \frac{\log \frac{y_k}{a_0}}{\log x_{ki}} \right\}$ is average of rates $\frac{\log \frac{y_k}{a_0}}{\log x_{ki}}$ larger than the overall average.

These reports fulfill the condition:

$$\frac{\log \frac{y_k}{a_0}}{\log x_{ki}} \geq \overline{\frac{\log \frac{y_k}{a_0}}{\log x_{ki}}}$$

The calculated parametres of the model are as follow:

p_1=0.214 p_2= 2.409 p_3=2.855 p_4=-0.193 p_5= 0.252

a_1=0.1151 a_2=0.5201 a_3=0.3983 a_4=0.0322 a_5= 0.0692

The multivariate model has the following appearance with the parameters calculated above:

p_1=0.214 p_2= 2.409 p_3=2.855 p_4=-0.193 p_5= 0.252

$Y=32.602x_1^{0.1151}\ x_2^{0.5201}x_3^{0.3983}\ x_4^{0.0322}x_5^{0.0692}$

Minimum cost and minimum potential was calculated with the formula:

$$z_{\min} = \frac{\sum_{i=1}^{n} c_i x_i}{y_{\max}} \quad \text{and} \quad z_{mund} = \frac{\sum_{i=1}^{n} c_i x_i}{y_{mund}},$$

where c_i prices for unit of inputs x_i (l/kv).

Based on these data, the following production functions were built:

$Y=32.602x_1^{0.1151}\ x_2^{0.5201}x_3^{0.3983}\ x_4^{0.0322}x_5^{0.0692}$

where x_1, x_2,x_3 , x_4, $x_5$and $y$show respectively the amount of inputs, manure, fertilizer, crystalline fertilizer, pesticides, watering and productivity.

3. Results and Discussion

The following function is given $y = a_0 x_1^{a_1} x_2^{a_2} x_3^{a_3} x_4^{a_4} x_5^{a_5}$

Lets sign r_1, r_2, r_3, r_4 and r_5 accordingly input prices x_1, x_2, x_3, x_4 dhe x_5 and then with p output price of y. If S is the set of all points $(x_1, x_2, x_3, x_4, x_5)$ that satisfy a number of equations we have the Lagrangean problem of maximizing or minimizing a function subject to equality constraints. Each solution $(x_1, x_2, x_3, x_4, x_5)$ gives a candidate
$(x_1, x_2, x_3, x_4, x_5)$ for an extreme point.
So, it is possible to compare the values of function at these candidate points to ascertain where its maximum and minimum values on S are attained.
We have formed Lagrange function LC for the minimum of cost

$$LC = r_1x_1 + r_2x_2 + r_3x_3 + r_4x_4 + r_5x_5 + \lambda[y - f(x_1, x_2, x_3, x_4, x_5)]$$

where λ is Lagrange multiplier, accompanied with the function of production y.
We have expressed the necessary conditions for the minimum of LC:

$$\begin{cases} \frac{\partial LC}{\partial x_1} = r_1 - \lambda f'_{x_1}(x_1, x_2, x_3, x_4, x_5) = 0 \\ \frac{\partial LC}{\partial x_2} = r_2 - \lambda f'_{x_2}(x_1, x_2, x_3, x_4, x_5) = 0 \\ \frac{\partial LC}{\partial x_3} = r_3 - \lambda f'_{x_3}(x_1, x_2, x_3, x_4, x_5) = 0 \\ \frac{\partial LC}{\partial x_4} = r_4 - \lambda f'_{x_4}(x_1, x_2, x_3, x_4, x_5) = 0 \\ \frac{\partial LC}{\partial x_5} = r_5 - \lambda f'_{x_5}(x_1, x_2, x_3, x_4, x_5) = 0 \\ \frac{\partial LC}{\partial \lambda} = y - f(x_1, x_2, x_3, x_4, x_5) = 0 \end{cases}$$

From this system we will have:

$$\begin{cases} r_1 - \lambda \frac{a_1 y}{x_1} = 0 \\ r_2 - \lambda \frac{a_2 y}{x_2} = 0 \\ r_3 - \lambda \frac{a_3 y}{x_3} = 0 \\ r_4 - \lambda \frac{a_4 y}{x_4} = 0 \\ r_5 - \lambda \frac{a_5 y}{x_5} = 0 \\ y - a_0 x_1^{a_1} x_2^{a_2} x_3^{a_3} x_4^{a_4} x_5^{a_5} = 0 \end{cases}$$

The bordered Hessian, a determinant that shall be denoted by $|\bar{H}|$, where the bar on top symbolizes the border.

$$|\bar{H}| = \begin{vmatrix} 0 & f'_{x_1} & f'_{x_2} & f'_{x_3} & f'_{x_4} & f'_{x_5} \\ f'_{x_1} & L''_{x_1x_1} & L''_{x_1x_2} & L''_{x_1x_3} & L''_{x_1x_4} & L''_{x_1x_5} \\ f'_{x_2} & L''_{x_2x_1} & L''_{x_2x_2} & L''_{x_2x_3} & L''_{x_2x_4} & L''_{x_2x_5} \\ f'_{x_3} & L''_{x_3x_1} & L''_{x_3x_2} & L''_{x_3x_3} & L''_{x_3x_4} & L''_{x_3x_5} \\ f'_{x_4} & L''_{x_4x_1} & L''_{x_4x_2} & L''_{x_4x_3} & L''_{x_4x_4} & L''_{x_4x_5} \\ f'_{x_5} & L''_{x_5x_1} & L''_{x_5x_2} & L''_{x_5x_3} & L''_{x_5x_4} & L''_{x_5x_5} \end{vmatrix} =$$

$$= \begin{vmatrix} 0 & 0.542316175 & 3.272393188 & 633.9855177 & 177.0581059 & 0.006439915 \\ 0.542316175 & 0.106996333 & 0 & 0 & 0 & 0 \\ 3.272393188 & 0 & 0.840172466 & 0 & 0 & 0 \\ 633.9855177 & 0 & 0 & 41178.70711 & 0 & 0 \\ 177.0581059 & 0 & 0 & 0 & 39728.31038 & 0 \\ 0.006439915 & 0 & 0 & 0 & 0 & 0.0000244556 \end{vmatrix}$$

$$= -99769.42102 < 0$$

We have proved that the cost function has a minimum for the values
$x_1 = 92$, $x_2 = 1.55$, $x_3 = 0.37$, $x_4 = 0.063$, $x_5 = 47.65$, after we have assessed the necessary conditions for the minimum through bordered Hessian determiner.

4. Conclusions

The study concluded with these results:
The study involved 16 communes in Lushnja region where the target group were farmers engaged in greenhouse tomato production. This region is the area with the greatest production of vegetables which supply the overwhelming part of the domestic retail market.
It was analyzedthe economic impact of five components (manure, fertilizer, crystalline manure, pesticides, irrigation) and productivity. In this studywas used Cobb-Douglas production function which gives the opportunity to build valuable economic analysis. By the use of this function was analyzed the influence of five inputs in one output. The values $\log a_0$, a_1, a_2, a_3, a_4 and a_5 were determined through the econometric computerized programmes, from which also resulted that the models were suitable. To realize the purpose of the study we built the Lagrange function (LC) to minimize the cost of tomato production. In conclusion, using the necessary conditions for the minimum of LC, it was found the optimal structure of the tomato production in this region.

5. References

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